Computers & Graphics 33 (2009) 147–150

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Chaos and Graphics

# Time-averaged patterns produced by stochastic moiré gratings

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ARTICLE INFO

*Keywords:*
Time-averaged moiré
Fringes
Random number generator

ABSTRACT

A simple computational algorithm is proposed for visualization of time-averaged moiré fringes. Time-averaging numerical techniques can be exploited for the construction of a cryptographic hash function. It appears that visualization of values of that hash function at continuously increasing amplitude of harmonic oscillation can produce interesting optical patterns. Chaotic and visually intriguing patterns are observed when the input of the hash function is selected as an array of random numbers uniformly distributed in the interval [0;1]. Such stochastic moiré gratings produce aesthetically beautiful pictures and can be used to explore aspects of the quality of a random number generator.



## 1. Introduction

Geometric moiré [1,2] patterns involve classical in-plane whole field non-destructive optical experimental techniques based on analysis of visual patterns produced by superposition of two regular gratings that geometrically interfere. Examples of gratings are equispaced parallel lines, concentric circles and arrays of dots. These gratings can be superposed by double-exposure photography, by reflection, by shadowing, or by direct contact [3,4]. Moiré patterns are used to measure variables such as displacements, rotations, curvature, and strain throughout the viewed area. In-plane moiré is typically conducted with gratings of equispaced, parallel lines [2,3].

We will concentrate only on a one-dimensional example. Moiré grating on the surface of a one-dimensional structure in the state of equilibrium can be interpreted as a periodic variation of black and white colors:

$$M_1(x) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\lambda}x\right); \tag{1}$$

where $x$ is the longitudinal coordinate; $M_1$ is the grayscale level of the surface at a point $x$; $\lambda$ is the pitch of the grating. A numerical value 0 of the function in the Eq. (1) corresponds to the black color; 1—to the white color; all intermediate values—to grayscale levels.

If the structure is deflected from the state of equilibrium by $u$, then the one-dimensional grating in the deformed state can be interpreted as follows:

$$M_2(x) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\lambda}(x-u)\right). \tag{2}$$

Additive superposition [5] of the original and the deformed gratings yields:

$$M_{12}(x,u) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\lambda}\left(1-\frac{u}{2x}\right)x\right)\cdot\cos\left(\frac{\pi u}{\lambda}\right). \tag{3}$$

If the deflection $u$ is now varied continuously, the effect of beatings will occur. Fringes will form at the centers of the beatings where

$$\cos\left(\frac{\pi u}{\lambda}\right) = 0. \tag{4}$$

The Eq. (4) can be used to construct an explicit relationship between the fringe order, the deflection $u$ and the pitch of the grating [1,2].

Double-exposure geometric moiré techniques can be extended to time-averaging geometric moiré methods when the moiré grating is formed on a surface of an elastic oscillating structure and time-averaging techniques are used for the registration of time-averaged patterns of fringes [5]. Time-averaging moiré is exploited in numerous engineering applications involving time-average projection, reflection, geometric moiré techniques [6–8]. Again, we will use a one-dimensional model to illustrate the formation of time-averaged fringes. We assume that the deflection from the state of equilibrium varies in time:

$$u(x,t) = a\sin(\omega t + \varphi), \tag{5}$$

where $\omega$ is the circular frequency, $\varphi$ is the phase and $a$ is the amplitude of oscillation. Then time-averaged grayscale level can be expressed in the following form [5,8]:

$$\begin{aligned} M_T(x) &= \lim_{T\to\infty}\frac{1}{T}\int_0^T\left(\frac{1}{2}+\frac{1}{2}\cos\left(\frac{2\pi}{\lambda}(x-a\sin(\omega t+\varphi))\right)\right)dt \\ &= \frac{1}{2}+\frac{1}{2}\cos\left(\frac{2\pi}{\lambda}x\right)J_0\left(\frac{2\pi}{\lambda}a\right), \end{aligned} \tag{6}$$

where $T$ is an exposure time; $J_0$ is the zero order Bessel function of the first kind. Time-averaged fringes will form at such amplitudes

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doi:10.1016/j.cag.2008.09.008

$a$ where $J_0(2\pi/\lambda a) = 0$. Now, the relationship between the fringe order, the deformation amplitude $a$ and the pitch of the grating takes the following form:

$$\frac{2\pi}{\lambda} a_i = r_i, \tag{7}$$

where $r_i$ denotes $i$-th root of the zero order Bessel function of the first kind; $a_i$ is the amplitude of oscillation at the center of the $i$-th fringe; and the fringe order is determined using automatic, semiautomatic or even manual fringe counting techniques [1] applied to an experimental pattern of fringes.

## 2. Visualization of time-averaged moiré fringes

Realistic visualization of optical fringes in digital environments is a demanding computational task. It usually involves a finite element dynamical model of an elastic deformable body, a geometric model of an optical setup comprising a virtual projection plane, a source of illumination and a point of observation, physical properties of the surface of the analyzed body, and finally, a digital model of interference effects taking place in the system under investigation [9,10].

We assume that the field of dynamic displacements does not depend on $x$ (Eq. (5)). Thus we will use previously described one-dimensional model instead of building a finite element model of an elastic deformable structure. Such an approach considerably simplifies computations but still allows illustrating the formation of double exposure and time-averaged moiré fringes. Moreover, one can exploit standard interpolation and visualization tools available in Matlab what makes the process even simpler. Code 1 shows example computer programming pseudocode.

CODE 1

```
% Plots time-averaged moire fringes              % remove all variables, globals, functions
    clear all

%——————————————————————————————                   % CONSTRUCTION OF THE MOIRE GRATING
step = 0.01;                                     % define the step of the mesh
MESH = 0:step:10;                                % form the mesh for the grating
lamda = 0.2;                                     % define the pitch of the grating
MOIRE = 0.5+0.5*cos(2*pi*MESH/lamda);            % form the grayscale moire grating

%——————————————————————————————                   % PREPARATIONS FOR AVERAGING IN TIME
amax = 1;                                        % define the maximum amplitude of oscillations
namp = 100;                                      % define the number of amplitude increments
astep = amax/step;                               % find how many steps fit into max amplitude
X = −amax:step:10+amax;                          % define the observation window
lengthx = length(X);                             % find the length of vector X
ntime = 128;                                     % define the number of iterations for
                                                 % averaging in time

%——————————————————————————————                   % AMPLITUDE LOOP
for i = 1:namp                                   %
    a = amax*(i-1)/namp;                         % calculate the current amplitude
    A(i) = a;                                    % save the value of the current amplitude
    SUM = zeros(size(X));                        % clear averaged grayscale levels

%——————————————————————————————                   % AVERAGING LOOP
for j = 1:ntime                                  %
    t = 2*pi*(j-1)/ntime;                        % calculate the current time
    IMG = ones(size(X));                         % the image is set to white color
    delta = a*cos(t)/step;                       % calculate the discrete deflection
    IMG(round(astep-delta)+1: ...                % the deflected moire grating ...
        round(lengthx-astep-delta))...           % is fitted into ...
         = MOIRE;                                % the current instantaneous image
        SUM = SUM+IMG;                           % add the current image
    end                                          % end of the AVERAGING LOOP
    SUM = SUM/ntime;                             % calculate averaged grayscale levels
    M(:,i) = SUM;                                % save the currently averaged image
end                                              % end of the AMPLITUDE LOOP

%——————————————————————————————                   % VISUALIZATION
pcolor(A,X,M)                                    % plot a colored parametric surface
                                                 % set the view to directly above
shading interp                                   % interpolate the shading
colormap gray                                    % set the grayscale colormap
```

The execution of this script produces a paradigm pattern of fringes resembling a time-averaged image of an elastic structure performing in-plane oscillations according to its first eigenmode with a horizontal moiré grating plotted on its surface (Fig. 1). Centers of time-averaged fringes are located at such amplitudes where the Eq. (7) holds true.

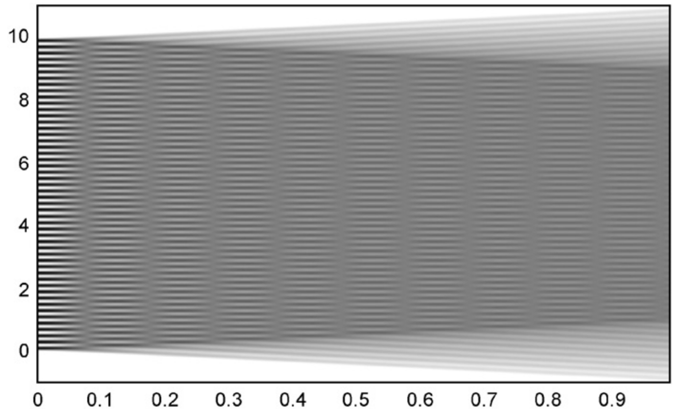


**Fig. 1.** A pattern of time-averaged fringes produced by the harmonic moiré grating.

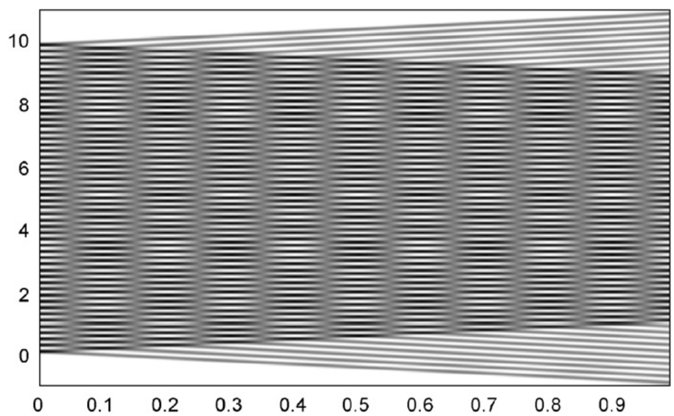


**Fig. 2.** A pattern of double exposure fringes produced by the harmonic moiré grating.

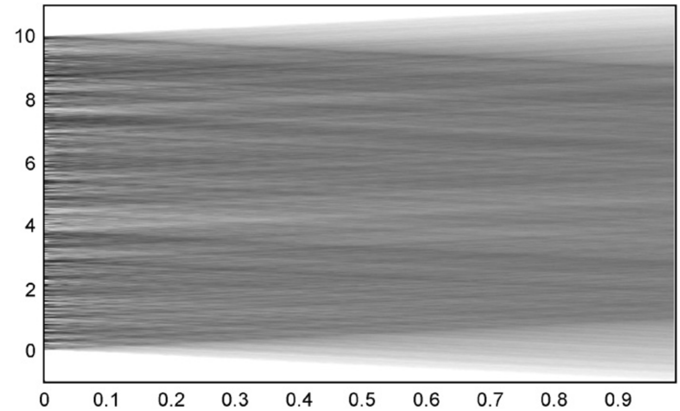


**Fig. 3.** A time-averaged image produced by a set of random numbers uniformly distributed in the interval [0;1].

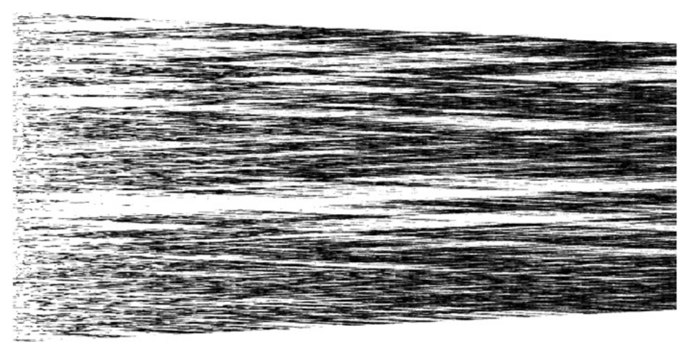

**Fig. 4.** A chaotic pattern of fringes produced from the time-averaged image by applying contrast enhancement techniques.

Fringes mimicking the experimental double exposure moiré technique can be plotted if the parameter ntime is changed from 128 to 2 (Fig. 2). Though the grayscale level at fringe centerlines is again 0.5, the variation of the grayscale level in regions between higher order fringes is not limited by the zero order Bessel function of the first kind (Eq. (3)). This is a nice example illustrating the necessity of contrast enhancement techniques for time-averaged moiré images.

## 3. Chaotic patterns produced by stochastic moiré gratings

An interesting time-averaged image is produced (Fig. 3) when the moiré grating is formed as a set of random numbers uniformly distributed in the interval [0;1]. This can be easily implemented into the script by changing the last two lines in the program section "CONSTRUCTION OF MOIRE GRATING" to:

```
for i = 1:length(MESH)
MOIRE(i) = rand; % form the grayscale moire grating
end
```

Such time-averaging of stochastic data is exploited for construction of a new class of hash functions [11]. The computational process embedded into the hash function [11] can be described mathematically by the following equation:

$$H_sF(x) = \Phi^{-1}(J_0(s\zeta)\Phi\tilde{F}(x)) + \tfrac{1}{2}, \quad (8)$$

where $F(x)$ is a grayscale function; $0 \leqslant F(x) \leqslant 1$; $H_s$ is a time-averaging operator; $s$ is the amplitude of harmonic oscillations; $\tilde{F}(x) = F(x) - 0.5$; $\Phi$ is the Fourrier transform; $\Phi^{-1}$ is the inverse Fourrier transform; $\zeta$ is the coordinate in the frequency domain.

The kernel of the operator described by the Eq. (8) is irregular because calculation of $F(x)$ (when $H_sF(x)$ is given) involves following computations:

$$\tilde{F}(x) = \Phi^{-1}\left(\frac{1}{J_0(s\zeta)}\Phi\left(H_sF(x) - \frac{1}{2}\right)\right). \quad (9)$$

But the zero order Bessel function of the first kind has multiple roots; therefore multiple divisions by zero occur in the process of the reconstruction of the original image. In other words, the inverse problem is ill-posed. Aiming at full recovery of the information results in unstable solutions due to the fact that the reconstructed image is very sensitive to inevitable measurement errors. In other words, slightly different data would produce a significantly different image.

Clearly, the object of this paper is not to concentrate on the details and properties of the hash function, but to present visually attractive chaotic fringes produced in the process of time-averaging of stochastic moiré gratings. In order to visualize this pattern of chaotic fringes we use contrast enhancement techniques mentioned in the previous section. The main idea of these contrast enhancement techniques is to map grayscale levels around 0.5 to the black color, and all other levels—to the white color. We use a hyperbolic tangent mapping function [12] to produce a contrast enhanced image presented in Fig. 4.

It is shown in [11] that for any grayscale function

$$\lim_{s\to\infty} H_s F(x) = O(x), \tag{10}$$

where $O(x)$ is the zero grayscale function; $O(x) = 0.5$ for all $x$. In other words, a contrast enhanced image will eventually become black at sufficiently large $s$. Such an effect is observed in Fig. 4. Nevertheless, the original set of random numbers undergoes extensive transformations before converging to 0.5. A unique fingerprint is produced in the process of these transformations. That fingerprint (besides an aesthetic value) can be used to explore aspects of the quality of a random number generator used to construct the original set of random numbers. Wide white streaks in Fig. 4 are a definite indicator that random numbers were not uniformly distributed in some parts of the original grayscale vector.

Graphical methods for assessing the quality of random number sequences are usually called Marsaglia plots [13]. As Vattulainen et al. showed some years ago, a carefully chosen 2-D test can be very revealing [14]. Of course, additional research needs to be conducted to more fully understand and assess the methods presented in this paper, and we look forward to hearing from readers who have explored the approach in a variety of experiments.

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